

## Grades K, 1 and 2

STANDARD 4.1 (NUMBER AND NUMERICAL OPERATIONS) ALL STUDENTS WILL DEVELOP NUMBER SENSE AND WILL PERFORM STANDARD NUMERICAL OPERATIONS AND ESTIMATIONS ON ALL TYPES OF NUMBERS IN A VARIETY OF WAYS.

**Descriptive Statement:** Numbers and arithmetic operations are what most of the general public think about when they think of mathematics; and, even though other areas like geometry, algebra, and data analysis have become increasingly important in recent years, numbers and operations remain at the heart of mathematical teaching and learning. Facility with numbers, the ability to choose the appropriate types of numbers and the appropriate operations for a given situation, and the ability to perform those operations as well as to estimate their results, are all skills that are essential for modern day life.

**Number Sense.** Number sense is an intuitive feel for numbers and a common sense approach to using them. It is a comfort with what numbers represent that comes from investigating their characteristics and using them in diverse situations. It involves an understanding of how different types of numbers, such as fractions and decimals, are related to each other, and how each can best be used to describe a particular situation. It subsumes the more traditional category of school mathematics curriculum called numeration and thus includes the important concepts of place value, number base, magnitude, and approximation and estimation.

**Numerical Operations.** Numerical operations are an essential part of the mathematics curriculum, especially in the elementary grades. Students must be able to select and apply various computational methods, including mental math, pencil-and-paper techniques, and the use of calculators. Students must understand how to add, subtract, multiply, and divide whole numbers, fractions, decimals, and other kinds of numbers. With the availability of calculators that perform these operations quickly and accurately, the instructional emphasis now is on understanding the meanings and uses of these operations, and on estimation and mental skills, rather than solely on the development of paper-and-pencil proficiency.

**Estimation.** Estimation is a process that is used constantly by mathematically capable adults, and one that can be easily mastered by children. It involves an educated guess about a quantity or an intelligent prediction of the outcome of a computation. The growing use of calculators makes it more important than ever that students know when a computed answer is reasonable; the best way to make that determination is through the use of strong estimation skills. Equally important is an awareness of the many situations in which an approximate answer is as good as, or even preferable to, an exact one. Students can learn to make these judgments and use mathematics more powerfully as a result.

Number and operation skills continue to be a critical piece of the school mathematics curriculum and, indeed, a very important part of mathematics. But, there is perhaps a greater need for us to rethink our approach here than to do so for any other curriculum component. An enlightened mathematics program for today's children will empower them to use all of today's tools rather than require them to meet yesterday's expectations.


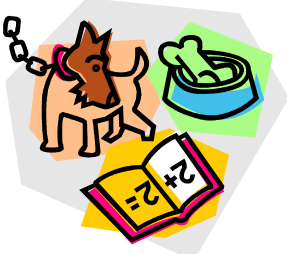
By the end of Grade 2, students will:

Number Sense


KINDERGARTEN	GRADE ONE	GRADE TWO
<ul style="list-style-type: none"> <li><input type="checkbox"/> Use real-life experiences, physical materials, and technology to construct meanings for numbers.</li> <li><input type="checkbox"/> Compare sets (more, less) up to <b>10</b> objects</li> <li><input type="checkbox"/> Identify <b>equivalent and non-equivalent sets</b></li> <li><input type="checkbox"/> Quickly see and label sets of objects up to 10</li> <li><input type="checkbox"/> Understand that numbers have a variety of uses</li> <li><input type="checkbox"/> Count money using coins</li> <li><input type="checkbox"/> Discuss Zero</li> <li><input type="checkbox"/> Interpret oral number sentences and represent them with concrete objects up to 10</li> <li><input type="checkbox"/> Verbally count forward and backward to and from 10</li> <li><input type="checkbox"/> Identify ordinals 1<sup>st</sup>s to 5<sup>th</sup></li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Use real-life experiences, physical materials, and technology to construct meanings for numbers.</li> <li><input type="checkbox"/> Construct and label sets of <b>20</b></li> <li><input type="checkbox"/> Recognize number words of sets through <b>20</b></li> <li><input type="checkbox"/> Identify number sense based on <b>100</b></li> <li><input type="checkbox"/> Understand <b>base ten</b> system: 10 ones = 1 ten; 10 tens = 100</li> <li><input type="checkbox"/> Count <b>items</b> up to 100</li> <li><input type="checkbox"/> Read and write numbers to <b>100</b></li> <li><input type="checkbox"/> Compare and order whole numbers up to 100</li> <li><input type="checkbox"/> Use the words higher, lower, greater, and less to compare two numbers and less to compare two numbers</li> <li><input type="checkbox"/> Count <b>money</b> using coins</li> <li><input type="checkbox"/> Quickly count by 1's to 100</li> <li><input type="checkbox"/> Quickly <b>skip count</b> by 10's to 100</li> <li><input type="checkbox"/> Quickly skip count by 5's to 50</li> <li><input type="checkbox"/> Quickly skip count by 2's to 20</li> <li><input type="checkbox"/> Count backwards from 20 to 1's</li> <li><input type="checkbox"/> Name the number before and after and between a given number</li> <li><input type="checkbox"/> Recognize <b>odd and even</b> numbers</li> <li><input type="checkbox"/> Identify and order <b>ordinal positions first to 10<sup>th</sup></b></li> <li><input type="checkbox"/> Explore the relationship between addition and subtraction using real objects</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Use real-life experiences, physical materials, and technology to construct meanings for numbers</li> <li><input type="checkbox"/> Identify number sense based on patterns to <b>1000</b></li> <li><input type="checkbox"/> Read and write and order numbers <b>words</b> to 20</li> <li><input type="checkbox"/> Read a number line to 100</li> <li><input type="checkbox"/> <b>Compare and order</b> numbers to 100</li> <li><input type="checkbox"/> Understand the <b>base ten system</b>: 10 ones = 1 ten; 10 tens = 1 hundred; 10 hundreds = 1000</li> <li><input type="checkbox"/> Recognize the meaning of <b>zero</b> in the place value system</li> <li><input type="checkbox"/> Determine <b>even and odd numbers</b></li> <li><input type="checkbox"/> Skip count to 100 by 2,5,10</li> <li><input type="checkbox"/> Count backward from 100 by 1,5,10</li> <li><input type="checkbox"/> Name the number before, after and between a given number</li> <li><input type="checkbox"/> Skip count by 3 to 36 for multiplication readiness</li> <li><input type="checkbox"/> Skip count by 4 to 48 for multiplication readiness</li> <li><input type="checkbox"/> Join and separate numbers</li> <li><input type="checkbox"/> Recognize the value of a <b>dollar</b> in relation to other coins</li> <li><input type="checkbox"/> Count money using coins and bills</li> <li><input type="checkbox"/> Identify and order <b>ordinals</b> positions first to <b>twentieth</b></li> </ul>



## Addition and Subtraction

KINDERGARTEN	GRADE ONE	GRADE TWO
<input type="checkbox"/> Solve and create many addition and subtraction <b>verbal word problems</b>  	<input type="checkbox"/> Demonstrate <b>accuracy and fluency</b> of addition and subtraction facts up to 12  	<input type="checkbox"/> By playing with numbers determine sums and differences of number sentences: Number families; related facts, inverse operations, addition doubles, doubles plus one <input type="checkbox"/> <b>Know with accuracy and fluency addition and subtraction facts up to and including 18</b> <input type="checkbox"/> <b>Do lots and lots of mental math</b> <input type="checkbox"/> Develop readiness for division by using repeated subtraction, dividing objects into groups

## Multiplication and Division

KINDERGARTEN	GRADE ONE	GRADE TWO
		<input type="checkbox"/> Demonstrate multiplication as repeated addition <input type="checkbox"/> Explore multiplication 2,3,4,5 using <b>real objects</b>

## Fractions

KINDERGARTEN	GRADE ONE	GRADE TWO
	<input type="checkbox"/> <b>Use real objects</b> to see proper fractions with denominators of 2,3,4,8,10	<input type="checkbox"/> <b>Use real objects</b> to see $\frac{1}{3}, \frac{2}{3}, \frac{3}{4}$ of a unite; $\frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}$

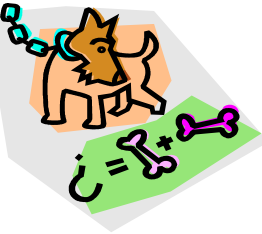
# NUMERICAL OPERATIONS

## ADDITION AND SUBTRACTION

KINDERGARTEN	GRADE ONE	GRADE TWO
<ul style="list-style-type: none"> <li><input type="checkbox"/> <b>Develop the meanings of addition and subtraction by concretely modeling and discussing a large variety of problems</b></li> <li><input type="checkbox"/> Check the reasonableness of results of computations</li> <li><input type="checkbox"/> <b>Order and write</b> 0-10</li> <li><input type="checkbox"/> Count down from 10</li> <li><input type="checkbox"/> Identify groups to 10</li> <li><input type="checkbox"/> Count by ones to 50</li> <li><input type="checkbox"/> Discover the meaning of addition and subtraction in real life situations</li> <li><input type="checkbox"/> Count and use concrete objects to explore addition and subtraction</li> <li><input type="checkbox"/> Apply fact families 1-5 for addition and subtraction with concrete objects</li> <li><input type="checkbox"/> Read and interpret a linear number line to 20</li> <li><input type="checkbox"/> Identify missing numbers on a number line up to 10 and in sequence up to 10</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> <b>Develop the meanings of addition and subtraction by concretely modeling and discussing a large variety of problems</b></li> <li><input type="checkbox"/> <b>In the context of real word problems solve:</b></li> <li><input type="checkbox"/> Addition and subtraction word problems without regrouping</li> <li><input type="checkbox"/> Understand and use the <b>inverse</b> relationship between addition and subtraction</li> <li><input type="checkbox"/> Check the reasonableness of results of computations</li> <li><input type="checkbox"/> <b>Read and write</b> in order numbers 0 -20</li> <li><input type="checkbox"/> Use both horizontal and vertical formats in addition and subtraction</li> <li><input type="checkbox"/> Insert correct operation signs (+ -) in vertical and horizontal number sentences</li> <li><input type="checkbox"/> Insert correct operation signs (+ -) in vertical and horizontal number sentences</li> <li><input type="checkbox"/> Identify and supply missing addends and subtrahends up to 12</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> <b>Develop the meanings of addition and subtraction by concretely modeling and discussing a large variety of problems</b></li> <li><input type="checkbox"/> <b>In the context of real word problems solve:</b></li> <li><input type="checkbox"/> Adding two digit numbers <b>without</b> regrouping</li> <li><input type="checkbox"/> Adding two digit numbers <b>with</b> regrouping</li> <li><input type="checkbox"/> Subtracting two digit numbers <b>without</b> regrouping</li> <li><input type="checkbox"/> Subtracting two digit numbers <b>with</b> regrouping</li> <li><input type="checkbox"/> Understand and use the <b>inverse</b> relationship between addition and subtraction</li> <li><input type="checkbox"/> Check the reasonableness of results of computations</li> <li><input type="checkbox"/> Construct, use and explain procedures for performing addition and subtraction calculations with pencil and paper. do lots of mental math</li> <li><input type="checkbox"/> Use dollar sign and cent sign correctly</li> </ul>



## Multiplication and Division

KINDERGARTEN	GRADE ONE	GRADE TWO
		<input type="checkbox"/> Explore the meanings of multiplication and division by modeling and discussing problems from the children's world

## Estimation

KINDERGARTEN	GRADE ONE	GRADE TWO
<input type="checkbox"/> Judge without counting whether a set of objects has less than, more than, or the same number of objects as a reference set <input type="checkbox"/> Determine the reasonableness of an answer by estimating the result of computations (e.g. $15+16$ is not 211) <input type="checkbox"/> Explore a variety of strategies for estimating both quantities (e.g. the number of marbles in a jar?) and results of computation jar and results of computation <input type="checkbox"/> Estimate numbers in a group <input type="checkbox"/> Estimate by visualizing which of two objects is taller <input type="checkbox"/> Estimate $\frac{1}{2}$ of an object	<input type="checkbox"/> Judge without counting whether a set of objects has less than, more than, or the same number of objects as a reference set. <input type="checkbox"/> Determine the reasonableness of an answer by estimating the result of computations (e.g. $15+16$ is not 211) <input type="checkbox"/> Explore a variety of strategies for estimating both quantities (e.g. the number of marbles in a jar?) and results of computation jar and results of computation <input type="checkbox"/> Determine when estimation is appropriate and explain the usefulness of an estimate as distinct form an exact answer <input type="checkbox"/> Estimate answers to computational type problems to determine reasonableness of an answer	<input type="checkbox"/> Judge without counting whether a set of objects has less than, more than, or the same number of objects as a reference set. <input type="checkbox"/> Determine the reasonableness of an answer by estimating the result of computations (e.g. $15+16$ is not 211) <input type="checkbox"/> Explore a variety of strategies for estimating both quantities (e.g. the number of marbles in a jar?) and results of computation jar and results of computation <input type="checkbox"/> Use concrete objects to estimate quantities of 10 and 100 using mental math <input type="checkbox"/> Determine when estimation is appropriate and understand the usefulness of an estimate as distinct from an exact answer <input type="checkbox"/> Estimate the reasonable amount of time needed to complete real world activities in hours and half-hours

STANDARD 4.2 (GEOMETRY AND MEASUREMENT) ALL STUDENTS WILL DEVELOP SPATIAL SENSE AND THE ABILITY TO USE GEOMETRIC PROPERTIES, RELATIONSHIPS, AND MEASUREMENT TO MODEL, DESCRIBE AND ANALYZE PHENOMENA.

**Descriptive Statement:** Spatial sense is an intuitive feel for shape and space. Geometry and measurement both involve describing the shapes we see all around us in art, nature, and the things we make. Spatial sense, geometric modeling, and measurement can help us to describe and interpret our physical environment and to solve problems.

**Geometric Properties.** This includes identifying, describing and classifying standard geometric objects, describing and comparing properties of geometric objects, making conjectures concerning them, and using reasoning and proof to verify or refute conjectures and theorems. Also included here are such concepts as symmetry, congruence, and similarity.

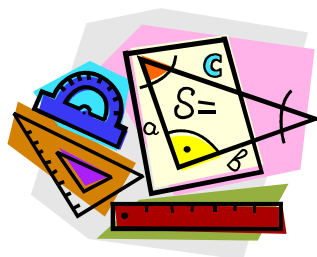
**Transforming Shapes.** Analyzing how various transformations affect geometric objects allows students to enhance their spatial sense. This includes combining shapes to form new ones and decomposing complex shapes into simpler ones. It includes the standard geometric transformations of translation (slide), reflection (flip), rotation (turn), and dilation (scaling). It also includes using tessellations and fractals to create geometric patterns.

**Coordinate Geometry.** Coordinate geometry provides an important connection between geometry and algebra. It facilitates the visualization of algebraic relationships, as well as an analytical understanding of geometry.

**Units of Measurement.** Measurement helps describe our world using numbers. An understanding of how we attach numbers to real-world phenomena, familiarity with common measurement units (e.g., inches, liters, and miles per hour), and a practical knowledge of measurement tools and techniques are critical for students' understanding of the world around them.

**Measuring Geometric Objects.** This area focuses on applying the knowledge and understandings of units of measurement in order to actually perform measurement. While students will eventually apply formulas, it is important that they develop and apply strategies that derive from their understanding of the attributes. In addition to measuring objects directly, students apply indirect measurement skills, using, for example, similar triangles and trigonometry.

Students of all ages should realize that geometry and measurement is all around them. Through study of these areas and their applications, they should come to better understand and appreciate the role of mathematics in their lives.



## Geometric Properties

KINDERGARTEN	GRADE ONE	GRADE TWO
<ul style="list-style-type: none"> <li><input type="checkbox"/> Identify and describe <b>spatial relationships</b> among object in space and their relative shapes and sizes. Inside/outside, left/right, above/below, between</li> <li><input type="checkbox"/> Use concrete objects, drawings and computer graphics to identify, classify and describe <b>standard 3D and 2D shapers:</b></li> <li><input type="checkbox"/> 3D figures – cube, rectangular prism, sphere, cone, cylinder and pyramid</li> <li><input type="checkbox"/> 2D figures – square, rectangle, circle, triangle</li> <li><input type="checkbox"/> Study the relationships between 3D and 2D shapes (e.g. the face of a 3D shape is a 2D shape)</li> <li><input type="checkbox"/> Identify and describe positions: top, bottom, above, below, over, under, up, down, high, low, before, after, first , last</li> <li><input type="checkbox"/> Determine if an object is bigger, smaller, longer or shorter than a given object</li> <li><input type="checkbox"/> Describe an object in location by using terms inside, outside, on...</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Identify and describe <b>spatial relationships</b> among object in space and their relative shapes and sizes. Inside/outside, left/right, above/below, between</li> <li><input type="checkbox"/> Use concrete objects, drawings and computer graphics to identify, classify and describe <b>standard 3D and 2D shapers:</b></li> <li><input type="checkbox"/> 3D figures – cube, rectangular prism, sphere, cone, cylinder and pyramid</li> <li><input type="checkbox"/> 2D figures – square, rectangle, circle, triangle</li> <li><input type="checkbox"/> Study the relationships between 3D and 2D shapes (e.g. the face of a 3D shape is a 2D shape)</li> <li><input type="checkbox"/> Describe, identify and create instances of <b>line symmetry</b></li> <li><input type="checkbox"/> Determine whether one in on/off/left/right, above/under/another object</li> <li><input type="checkbox"/> Using concrete objects determine whether two objects are larger/smaller where shape does not matter</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Identify and describe spatial relationships among object in space and their relative shapes and sizes. Inside/outside, left/right, above/below, between</li> <li><input type="checkbox"/> Study objects that have <b>congruence</b> (e.g. same size and shape)</li> <li><input type="checkbox"/> I Use concrete objects, drawings and computer graphics to identify, classify and describe <b>standard 3D and 2D shapers:</b></li> <li><input type="checkbox"/> 3D figures – cube, rectangular prism, sphere, cone, cylinder and pyramid</li> <li><input type="checkbox"/> 2D figures – square, rectangle, circle, triangle</li> <li><input type="checkbox"/> Find on objects and name the parts: Vertex, edge, face, side</li> <li><input type="checkbox"/> Study the relationships between 3D and 2D shapes (e.g. the face of a 3D shape is a 2D shape)</li> <li><input type="checkbox"/> Describe, identify and create instances of <b>line symmetry</b></li> <li><input type="checkbox"/> Identify, describe, match the name and classify <b>solid/space geometric figures</b> ( sphere, cone, cylinder, cube, rectangular, prism and pyramid)</li> </ul>

## Geometry

KINDERGARTEN	GRADE ONE	GRADE TWO
<ul style="list-style-type: none"> <li><input type="checkbox"/> Identify and illustrate <b>turns and sides</b> using body movements and manipulatives</li> <li><input type="checkbox"/> Identify a <b>mirror image as a flip</b></li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Compare <b>symmetry and congruence</b></li> <li><input type="checkbox"/> Draw congruent shapes</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Compare symmetry and congruence</li> <li><input type="checkbox"/> Identify flip images</li> <li><input type="checkbox"/> Draw congruent shapes</li> </ul>

## Transforming Shapes

KINDERGARTEN	GRADE ONE	GRADE TWO
<ul style="list-style-type: none"> <li><input type="checkbox"/> Use simple shapes to make <b>designs, patterns and pictures</b></li> <li><input type="checkbox"/> <b>Combine and subdivide</b> simple shapes to make other shapes</li> <li><input type="checkbox"/> Identify circles, squares, triangles as they occur in nature</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Use simple shapes to make designs, patterns and pictures</li> <li><input type="checkbox"/> Combine and subdivide simple shapes to make other shapes</li> <li><input type="checkbox"/> <b>Predict</b> outcome of putting two shapes together</li> <li><input type="checkbox"/> Determine congruence and similarity of various objects in different positions</li> <li><input type="checkbox"/> Perform a left/right turn as a preface to right angles</li> <li><input type="checkbox"/> Determine if a positional change is a flip, slide or turn</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Use simple shapes to make designs, patterns and pictures</li> <li><input type="checkbox"/> Combine and subdivide simple shapes to make other shapes</li> <li><input type="checkbox"/> Create two congruent figures</li> <li><input type="checkbox"/> Subdivide given shapes into other shapes</li> <li><input type="checkbox"/> Identify and extend a pattern as a flip, a slide or a turn</li> </ul>


## Coordinate Geometry

KINDERGARTEN	GRADE ONE	GRADE TWO
<ul style="list-style-type: none"> <li><input type="checkbox"/> Give and follow directions for getting from one point to another on a map or grid</li> <li><input type="checkbox"/> Identify an open and closed path</li> <li><input type="checkbox"/> Create a number path by following dots to a specific location</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Give and follow directions for getting from one point to another on a map or grid</li> <li><input type="checkbox"/> If given two locations, tell how to get from one to another; transform this to a simple grid</li> <li><input type="checkbox"/> Create a number path by following dots to a specific location</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Give and follow directions for getting from one point to another on a map or grid</li> <li><input type="checkbox"/> Identify location and direction using a map grid</li> <li><input type="checkbox"/> Create a number path by following dots to a specific location</li> </ul>

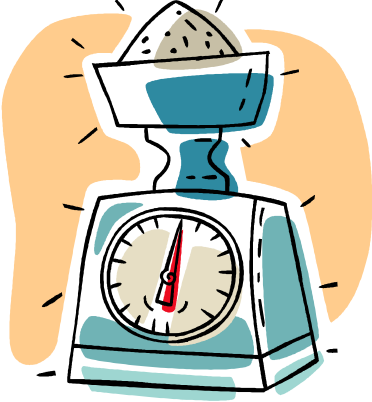
## Units of Measurement

KINDERGARTEN	GRADE ONE	GRADE TWO
<ul style="list-style-type: none"> <li><input type="checkbox"/> Directly compare and order objects according to measurable attributes</li> <li><input type="checkbox"/> Attributes – length, weight, capacity, time, temperature</li> <li><input type="checkbox"/> Recognize the need for a uniform unit of measure</li> <li><input type="checkbox"/> Select and use appropriate standard and non-standard units of measure and standard measurement tools to solve real-life problems</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Directly compare and order objects according to measurable attributes</li> <li><input type="checkbox"/> Attributes – length, weight, capacity, time, temperature</li> <li><input type="checkbox"/> Recognize the need for a uniform unit of measure</li> <li><input type="checkbox"/> Select and use appropriate standard and non-standard units of measure and standard measurement tools to solve real-life problems</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Directly compare and order objects according to measurable attributes</li> <li><input type="checkbox"/> Attributes – length, weight, capacity, time, temperature</li> <li><input type="checkbox"/> Recognize the need for a uniform unit of measure</li> <li><input type="checkbox"/> Select and use appropriate standard and non-standard units of measure and standard measurement tools to solve real-life problems</li> </ul>

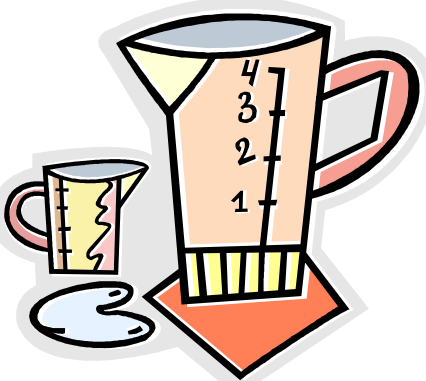
## Units of Measurements: Lengths

KINDERGARTEN	GRADE ONE	GRADE TWO
<ul style="list-style-type: none"> <li><input type="checkbox"/> Estimate the measure by using personal referent such as width of a finger, thumb, foot as a unit of measure</li> </ul> 	<ul style="list-style-type: none"> <li><input type="checkbox"/> Use a ruler to measure <b>inches</b> to 12 for length</li> <li><input type="checkbox"/> Use a ruler to measure <b>centimeters</b> to 20</li> <li><input type="checkbox"/> Use non-standard units to compare height and length</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Use customary and metric units to measure length to the nearest <b>inch, half-inch, foot, centimeter and meter</b></li> <li><input type="checkbox"/> Compare measurement (in length of objects using non-standard units to measurement of same objects using standard units)</li> <li><input type="checkbox"/> Use personal body referents (baby steps/giant steps) to determine the larger the unit, the smaller the measure, the smaller the unit, the larger the measure</li> </ul>

## Units of Measurement: Weight

KINDERGARTEN	GRADE ONE	GRADE TWO
	<input type="checkbox"/> Use a balance scale to measure a <b>pound</b>	<input type="checkbox"/> Use a balance scale to measure <b>grams, kilograms, pounds and ounces</b>



## Units of Measurements: Capacity

KINDERGARTEN	GRADE ONE	GRADE TWO
	<input type="checkbox"/> Recognize a cup, pint, quart, gallon	<input type="checkbox"/> Measure capacity in cups, pints, quarts, gallons and liters <input type="checkbox"/> Measure in <b>pounds, ounces, kilograms and grams</b> <input type="checkbox"/> Convert cups to pints and quarts <input type="checkbox"/> Convert cups to pints, pints to quarts, quarts to gallons

## Units of Measurements: Temperature

KINDERGARTEN	GRADE ONE	GRADE TWO
<input type="checkbox"/> Describe temperature as hot or cold on a thermometer	<input type="checkbox"/> Judge whether the temperatures are hot, warm, cool and cold	<input type="checkbox"/> Read temperature on a thermometer in both <b>Fahrenheit and Celsius</b> degrees within a five day range



## Units of Measurements: Money

KINDERGARTEN	GRADE ONE	GRADE TWO
<ul style="list-style-type: none"> <li><input type="checkbox"/> Identify the cent sign</li> <li><input type="checkbox"/> Identify the coins: Penny, nickel, dime</li> <li><input type="checkbox"/> Count to ten cents</li> </ul> 	<ul style="list-style-type: none"> <li><input type="checkbox"/> Identify and compare penny, nickel, dime, quarter and dollar</li> <li><input type="checkbox"/> Count and make change to \$1.0</li> <li><input type="checkbox"/> Add and subtract money</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Convert money: Pennies to nickel; nickel to dime; dime to ½ dollar...</li> </ul> 

## Units of Measurements: Time

KINDERGARTEN	GRADE ONE	GRADE TWO
<ul style="list-style-type: none"> <li><input type="checkbox"/> Describe a short time and a long time</li> <li><input type="checkbox"/> Tell time to the hour</li> <li><input type="checkbox"/> Use a calendar for the months, weeks, seasons of the year</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Identify days of the week, months of the year, seasons</li> <li><input type="checkbox"/> Tell time on both a digital and analog clock to the <b>hour and half hour</b></li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Show time to the hour, half hour, quarter hour and five minute intervals on both digital and analog clocks.</li> </ul>

## Measuring Geometric Objects

KINDERGARTEN	GRADE ONE	GRADE TWO
	<ul style="list-style-type: none"> <li><input type="checkbox"/> Directly measure the <b>perimeter</b> of simple two-dimensional shapes</li> <li><input type="checkbox"/> Describe different ways to measure two dimensional objects</li> <li><input type="checkbox"/> Find <b>area</b> of items using informal measurement instruments</li> </ul> 	<ul style="list-style-type: none"> <li><input type="checkbox"/> Directly measure the perimeter and <b>area</b> of simple two-dimensional objects. Use tiles for area</li> <li><input type="checkbox"/> Describe different ways to measure two and three dimensional objects</li> <li><input type="checkbox"/> Find area of two and three dimensional objects</li> <li><input type="checkbox"/> Estimate answers to computational type problems to determine reasonableness of answer</li> <li><input type="checkbox"/> Estimate <b>length, width and height</b> using informal measurements</li> <li><input type="checkbox"/> Estimate <b>volume</b> of containers using informal measurements</li> <li><input type="checkbox"/> Estimate length in feet and meters; <b>capacity</b> in pints, quarts, gallons, liter; weight in pounds and kilograms</li> <li><input type="checkbox"/> Estimate and measure the capacity of a larger container in terms of a smaller container</li> <li><input type="checkbox"/> Use the guess and check strategy to estimate the number of one inch squares to fill an area, the number of one inch cubes to fill a container, the number of edges of one inch squares to measure perimeter</li> </ul>

**STANDARD 4.3 (PATTERNS AND ALGEBRA) ALL STUDENTS WILL REPRESENT AND ANALYZE RELATIONSHIPS AMONG VARIABLE QUANTITIES AND SOLVE PROBLEMS INVOLVING PATTERNS, FUNCTIONS, AND ALGEBRAIC CONCEPTS AND PROCESSES.**

**Descriptive Statement:** Algebra is a symbolic language used to express mathematical relationships. Students need to understand how quantities are related to one another, and how algebra can be used to concisely express and analyze those relationships. Modern technology provides tools for supplementing the traditional focus on algebraic procedures, such as solving equations, with a more visual perspective, with graphs of equations displayed on a screen. Students can then focus on understanding the relationship between the equation and the graph, and on what the graph represents in a real-life situation.

**Patterns.** Algebra provides the language through which we communicate the patterns in mathematics. From the earliest age, students should be encouraged to investigate the patterns that they find in numbers, shapes, and expressions, and, by doing so, to make mathematical discoveries. They should have opportunities to analyze, extend, and create a variety of patterns and to use pattern-based thinking to understand and represent mathematical and other real-world phenomena.

**Functions and Relationships.** The function concept is one of the **most fundamental** unifying ideas of modern mathematics. Students begin their study of functions in the primary grades, as they observe and study patterns. As students grow and their ability to abstract matures, students form rules, display information in a table or chart, and write equations, which express the relationships they have observed. In high school, they use the more formal language of algebra to describe these relationships.

**Modeling.** Algebra is used to model real situations and answer questions about them. This use of algebra requires the ability to represent data in tables, pictures, graphs, equations or inequalities, and rules. Modeling ranges from writing simple number sentences to help solve story problems in the primary grades to using functions to describe the relationship between two variables, such as the height of a pitched ball over time. Modeling also includes some of the conceptual building blocks of calculus, such as how quantities change over time and what happens in the long run (limits).

**Procedures.** Techniques for manipulating algebraic expressions – procedures – remain important, especially for students who may continue their study of mathematics in a calculus program. Utilization of algebraic procedures includes understanding and applying properties of numbers and operations, using symbols and variables appropriately, working with expressions, equations, and inequalities, and solving equations and inequalities.

Algebra is a gatekeeper for the future study of mathematics, science, the social sciences, business, and a host of other areas. In the past, algebra has served as a filter, screening people out of these opportunities. For New Jersey to be part of the global society, it is important that algebra play a major role in a mathematics program that opens the gates for all students.

## Patterns

KINDERGARTEN	GRADE ONE	GRADE TWO
<input type="checkbox"/> Recognize, describe, extend and create patterns <input type="checkbox"/> Use concrete materials (manipulatives), pictures, rhythms and whole numbers create patterns <input type="checkbox"/> Use words and symbols (e.g., “add two” or +2) to create patterns <input type="checkbox"/> Make repeating patterns  <input type="checkbox"/> Explore whole number patterns that grow or shrink as a result if repeatedly adding or subtracting a fixed number (e.g. skip counting forward or backward)  <input type="checkbox"/> Find and describe patterns in real life  <input type="checkbox"/> Copy simple patterns using concrete materials  <input type="checkbox"/> Continue given patterns using concrete materials  <input type="checkbox"/> Sort, copy and continue simple patterns involving shapes, colors or sizes	<input type="checkbox"/> Recognize, describe, extend and create patterns <input type="checkbox"/> Use concrete materials (manipulatives), pictures, rhythms and whole numbers create patterns <input type="checkbox"/> Use words and symbols (e.g., “add two” or +2) to create patterns <input type="checkbox"/> Make repeating patterns <input type="checkbox"/> Explore whole number patterns that grow or shrink as a result if repeatedly adding or subtracting a fixed number (e.g. skip counting forward or backward) <input type="checkbox"/> Find and describe patterns in real life <input type="checkbox"/> Copy simple patterns using concrete materials <input type="checkbox"/> Continue given patterns using concrete materials <input type="checkbox"/> Continue and extend patterns forward, backward, horizontally, and vertically. (1,2,3,4;4,3,2,1;Sunday, Monday, Tuesday, Saturday, Friday, Thursday) <input type="checkbox"/> Identify missing numbers in sequence up to 100 <input type="checkbox"/> Discover pattern of change in a given situation and tell the rule <input type="checkbox"/> Discover pattern of change in a given situation and tell the rule <input type="checkbox"/> Create complex patterns using more than one attribute <input type="checkbox"/> Find patterns of change in growth	<input type="checkbox"/> Recognize, describe, extend and create patterns <input type="checkbox"/> Use concrete materials (manipulatives), pictures, rhythms and whole numbers create patterns <input type="checkbox"/> Use words and symbols (e.g., “add two” or +2) to create patterns <input type="checkbox"/> Make repeating patterns <input type="checkbox"/> Explore whole number patterns that grow or shrink as a result if repeatedly adding or subtracting a fixed number (e.g. skip counting forward or backward) <input type="checkbox"/> Find and describe patterns in real life <input type="checkbox"/> Copy simple patterns using concrete materials <input type="checkbox"/> Continue given patterns using concrete materials <input type="checkbox"/> Discover number patterns using the 100 chart counting by 2,5,20 <input type="checkbox"/> Identify, complete and create number patterns in addition and subtraction <input type="checkbox"/> Identify missing numbers in sequences to 100 (mentally) and to 500 using a chart  <input type="checkbox"/> Create patterns with two attributes using concrete patterns <input type="checkbox"/> Classify objects and continue patterns using two patterns <input type="checkbox"/> Record data collected on patterns observed over periods of time

## Functions and Relationships

KINDERGARTEN	GRADE ONE	GRADE TWO
<input type="checkbox"/> Use concrete and pictorial models of function machines to explore the basic concept of a function (In/Out)	<input type="checkbox"/> Use concrete and pictorial models of function machines to explore the basic concept of a function (In/Out)	<input type="checkbox"/> Use concrete and pictorial models of function machines to explore the basic concept of a function (In/Out)

## Modeling

KINDERGARTEN	GRADE ONE	GRADE TWO
<input type="checkbox"/> Recognize and describe changes over time (e.g. temperature, height) <input type="checkbox"/> Construct and solve simple open sentences involving addition or subtraction ( $2+_ =4$ )	<input type="checkbox"/> Recognize and describe changes over time (e.g. temperature, height) <input type="checkbox"/> Construct and solve simple open sentences involving addition or subtraction ( $8+_ =16$ ; $n-4=$ )	<input type="checkbox"/> Recognize and describe changes over time (e.g. temperature, height) <input type="checkbox"/> Construct and solve simple open sentences involving addition or subtraction ( $16-n=7$ ; $18-9=n$ )

## Procedures

KINDERGARTEN	GRADE ONE	GRADE TWO
<input type="checkbox"/> Understand and apply (but don't name) the following properties of addition: <input type="checkbox"/> Commutative (e.g. $5+3=3+5$ )  <input type="checkbox"/> Zero as the identity element (e.g. $7+0=7$ )  <input type="checkbox"/> Associative (e.g. $7+3+2$ can be found by first adding either $7+3$ or $3+2$ )	<input type="checkbox"/> Understand and apply (but don't name) the following properties of addition: <input type="checkbox"/> Commutative (e.g. $5+3=3+5$ )  <input type="checkbox"/> Zero as the identity element (e.g. $7+0=7$ )  <input type="checkbox"/> Associative (e.g. $7+3+2$ can be found by first adding either $7+3$ or $3+2$ )  <input type="checkbox"/> Use symbols $><$ and $=$ when comparing given numbers to 100  <input type="checkbox"/> Compare sets $<=>$	<input type="checkbox"/> Understand and apply (but don't name) the following properties of addition:  <input type="checkbox"/> Commutative (e.g. $5+3=3+5$ )  <input type="checkbox"/> Zero as the identity element (e.g. $7+0=7$ )  <input type="checkbox"/> Associative (e.g. $7+3+2$ can be found by first adding either $7+3$ or $3+2$ )  <input type="checkbox"/> Use symbols $><$ and $=$ when comparing given numbers to 100  <input type="checkbox"/> Use commutative property for addition

**STANDARD 4.4 (DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS) ALL STUDENTS WILL DEVELOP AN UNDERSTANDING OF THE CONCEPTS AND TECHNIQUES OF DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS, AND WILL USE THEM TO MODEL SITUATIONS, SOLVE PROBLEMS, AND ANALYZE AND DRAW APPROPRIATE INFERENCES FROM DATA.**

**Descriptive Statement:** Data analysis, probability, and discrete mathematics are important interrelated areas of applied mathematics. Each provides students with powerful mathematical perspectives on everyday phenomena and with important examples of how mathematics is used in the modern world. Two important areas of discrete mathematics are addressed in this standard; a third area, iteration and recursion, is addressed in Standard 4.3 (Patterns and Algebra).

**Data Analysis (or Statistics).** In today's information-based world, students need to be able to read, understand, and interpret data in order to make informed decisions. In the early grades, students should be involved in collecting and organizing data, and in presenting it using tables, charts, and graphs. As they progress, they should gather data using sampling, and should increasingly be expected to analyze and make inferences from data, as well as to analyze data and inferences made by others.

**Probability.** Students need to understand the fundamental concepts of probability so that they can interpret weather forecasts, avoid unfair games of chance, and make informed decisions about medical treatments whose success rate is provided in terms of percentages. They should regularly be engaged in predicting and determining probabilities, often based on experiments (like flipping a coin 100 times), but eventually based on theoretical discussions of probability that make use of systematic counting strategies. High school students should use probability models and solve problems involving compound events and sampling.

**Discrete Mathematics—Systematic Listing and Counting.** Development of strategies for listing and counting can progress through all grade levels, with middle and high school students using the strategies to solve problems in probability. Primary students, for example, might find all outfits that can be worn using two coats and three hats; middle school students might systematically list and count the number of routes from one site on a map to another; and high school students might determine the number of three-person delegations that can be selected from their class to visit the mayor.

**Discrete Mathematics—Vertex-Edge Graphs and Algorithms.** Vertex-edge graphs, consisting of dots (vertices) and lines joining them (edges), can be used to represent and solve problems based on real-world situations. Students should learn to follow and devise lists of instructions, called "algorithms," and use algorithmic thinking to find the best solution to problems like those involving vertex-edge graphs, but also to solve other problems.

These topics provide students with insight into how mathematics is used by decision-makers in our society, and with important tools for modeling a variety of real-world situations. Students will better understand and interpret the vast amounts of quantitative data that they are exposed to daily, and they will be able to judge the validity of data-supported arguments.

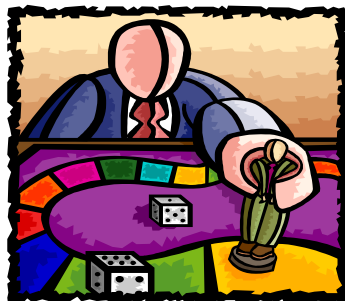
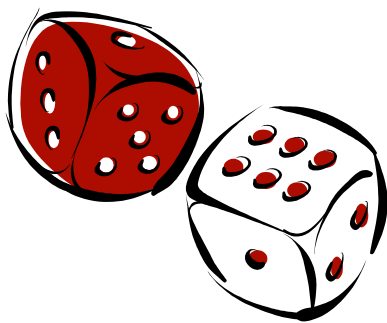
## Date Analysis

KINDERGARTEN	GRADE ONE	GRADE TWO
<input type="checkbox"/> Collect, generate and organize data in response to questions, claims, or curiosity <input type="checkbox"/> Date collected from student's everyday experiences <input type="checkbox"/> Data generated from chance devices, such as spinners and dice  <input type="checkbox"/> Read, interpret, construct, and analyze displays of data  <input type="checkbox"/> Pictures, tally chart, pictograph, bar graph, Venn diagram  <input type="checkbox"/> Smallest to largest, most frequent (mode)  <input type="checkbox"/> Gather data relating to familiar experiences by counting and tallying  <input type="checkbox"/> Draw conclusions based on data displayed on graphs	<input type="checkbox"/> Collect, generate and organize data in response to questions, claims, or curiosity <input type="checkbox"/> Date collected from student's everyday experiences <input type="checkbox"/> Data generated from chance devices, such as spinners and dice <input type="checkbox"/> Read, interpret, construct, and analyze displays of data  <input type="checkbox"/> Pictures, tally chart, pictograph, bar graph, Venn diagram <input type="checkbox"/> Smallest to largest, most frequent (mode)  <input type="checkbox"/> Collect and organize information; tally the results and record them on a graph  <input type="checkbox"/> Read and interpret graphs using symbols  <input type="checkbox"/> construct, read and interpret displays of data: pictographs, bar graphs, tables, lists	<input type="checkbox"/> Collect, generate and organize data in response to questions, claims, or curiosity  <input type="checkbox"/> Date collected from student's everyday experiences <input type="checkbox"/> Data generated from chance devices, such as spinners and dice  <input type="checkbox"/> Read, interpret, construct, and analyze displays of data  <input type="checkbox"/> Pictures, tally chart, pictograph, bar graph, Venn diagram  <input type="checkbox"/> Smallest to largest, most frequent (mode)  <input type="checkbox"/> Take a survey, tally and graph the results using familiar topics <input type="checkbox"/> Interpret compile pictographs, horizontal and vertical bar graphs from the survey  <input type="checkbox"/> Create tables to describe patterns, record data on the patterns observed over a period of time




## Probability

KINDERGARTEN	GRADE ONE	GRADE TWO
<ul style="list-style-type: none"> <li><input type="checkbox"/> Use chance devices like spinners and dice to explore concepts of probability: Certain, impossible more likely, less likely, equally likely</li> <li><input type="checkbox"/> Provide probability of specific outcomes</li> <li><input type="checkbox"/> Probability of getting specific outcome when coin is tossed, when die is rolled, when spinner is spun (e.g. if spinner has five equal sectors, then probability of getting a particular sector is one out of five)</li> <li><input type="checkbox"/> When picking a marble from a bag with tree red marbles and four blue marbles, the probability of getting a red marble is three out of seven</li>   <li><input type="checkbox"/> Perform and record the <b>outcome</b> of a simple probability activity</li> <li><input type="checkbox"/> Make <b>predictions</b> based on real-life experiences</li> <li><input type="checkbox"/> Use chance devices like spinners and dice to explore concepts of probability</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Use chance devices like spinners and dice to explore concepts of probability: Certain, impossible more likely, less likely, equally likely</li> <li><input type="checkbox"/> Provide probability of specific outcomes</li> <li><input type="checkbox"/> Probability of getting specific outcome when coin is tossed, when die is rolled, when spinner is spun (e.g. if spinner has five equal sectors, then probability of getting a particular sector is one out of five)</li> <li><input type="checkbox"/> When picking a marble from a bag with tree red marbles and four blue marbles, the probability of getting a red marble is three out of seven</li>   <li><input type="checkbox"/> Make a prediction of the outcome of a sample probability activity</li> <li><input type="checkbox"/> Perform a <b>sample probability</b> activity</li> <li><input type="checkbox"/> Use real-life situations to make predictions and conclusions based on experiences</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Use chance devices like spinners and dice to explore concepts of probability: Certain, impossible more likely, less likely, equally likely</li> <li><input type="checkbox"/> Provide probability of specific outcomes</li>   <li><input type="checkbox"/> Probability of getting specific outcome when coin is tossed, when die is rolled, when spinner is spun (e.g. if spinner has five equal sectors, then probability of getting a particular sector is one out of five)</li> <li><input type="checkbox"/> When picking a marble from a bag with tree red marbles and four blue marbles, the probability of getting a red marble is three out of seven</li>   <li><input type="checkbox"/> Use real-life situations to make predictions and draw conclusions</li>   <li><input type="checkbox"/> <b>Collect and organize</b> data, tally results and make a bar graph. Use the information to make predictions</li> <li><input type="checkbox"/> Perform simple probability activities and discuss possible number of times a particular result might occur</li>   <li><input type="checkbox"/> Make predictions of the outcomes of simple probability activities, experiments, tally results and compare with predictions</li> <li><input type="checkbox"/> Predict outcome of putting several shapes together</li> </ul>



## Discrete Mathematics—Systematic Listing and Counting

KINDERGARTEN	GRADE ONE	GRADE TWO
<ul style="list-style-type: none"> <li><input type="checkbox"/> Sort and classify objects according to attributes</li> <li><input type="checkbox"/> Use Venn diagrams to show number facts and families</li> <li><input type="checkbox"/> Generate all possibilities in simple counting situation (e.g. all outfits involving two shirts and three pants)</li> <li><input type="checkbox"/> Systemic listening and counting – sort and classify objects according to attributes</li> <li><input type="checkbox"/> Explore possibilities in simple counting situations (all outfits, involving two shirts and three pants)</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Sort and classify objects according to attributes</li> <li><input type="checkbox"/> Use Venn diagrams to show number facts and families</li> <li><input type="checkbox"/> Generate all possibilities in simple counting situation (e.g. all outfits involving two shirts and three pants)</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Sort and classify objects according to attributes</li> <li><input type="checkbox"/> Use Venn diagrams to show number facts and families</li> <li><input type="checkbox"/> <b>Generate all</b> possibilities in simple counting situation (e.g. all outfits involving two shirts and three pants)</li> </ul> <div style="text-align: center; margin-top: 20px;">  </div>

## Discrete Mathematics—Vertex-Edge Graphs and Algorithms

KINDERGARTEN	GRADE ONE	GRADE TWO
<ul style="list-style-type: none"> <li><input type="checkbox"/> Follow simple sets of directions (e.g. from one location to another or from a recipe)</li> <li><input type="checkbox"/> Color simple maps with a small number of colors</li> <li><input type="checkbox"/> Play simple two-person games (e.g. tic-tac-toe) and informally explore the idea of what the outcome should be</li> <li><input type="checkbox"/> Explore concrete models of vertex-edge graphs (e.g. vertices as “island” and edges as “bridges”)</li> <li><input type="checkbox"/> Paths from one vertex to another</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Follow simple sets of directions (e.g. from one location to another or from a recipe)</li> <li><input type="checkbox"/> Color simple maps with a small number of colors</li> <li><input type="checkbox"/> Play simple two-person games (e.g. tic-tac-toe) and informally explore the idea of what the outcome should be</li> <li><input type="checkbox"/> Explore concrete models of vertex-edge graphs (e.g. vertices as “island” and edges as “bridges”)</li> <li><input type="checkbox"/> Paths from one vertex to another</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Follow simple sets of directions (e.g. from one location to another or from a recipe)</li> <li><input type="checkbox"/> Color simple maps with a small number of colors</li> <li><input type="checkbox"/> Play simple two-person games (e.g. tic-tac-toe) and informally explore the idea of what the outcome should be</li> <li><input type="checkbox"/> Explore concrete models of vertex-edge graphs (e.g. vertices as “island” and edges as “bridges”)</li> <li><input type="checkbox"/> Paths from one vertex to another</li> </ul>

**STANDARD 4.5 (MATHEMATICAL PROCESSES) ALL STUDENTS WILL USE MATHEMATICAL PROCESSES OF PROBLEM SOLVING, COMMUNICATION, CONNECTIONS, REASONING, REPRESENTATIONS, AND TECHNOLOGY TO SOLVE PROBLEMS AND COMMUNICATE MATHEMATICAL IDEAS.**

**Descriptive Statement:** The mathematical processes described here highlight ways of acquiring and using the content knowledge and skills delineated in the first four mathematics standards.

**Problem Solving.** Problem posing and problem solving involve examining situations that arise in mathematics and other disciplines and in common experiences, describing these situations mathematically, formulating appropriate mathematical questions, and using a variety of strategies to find solutions. Through problem solving, students experience the power and usefulness of mathematics. Problem solving is interwoven throughout the grades to provide a context for learning and applying mathematical ideas.

**Communication.** Communication of mathematical ideas involves students' sharing their mathematical understandings in oral and written form with their classmates, teachers, and parents. Such communication helps students clarify and solidify their understanding of mathematics and develop confidence in them as mathematics learners. It also enables teachers to better monitor student progress.

**Connections.** Making connections involves seeing relationships between different topics, and drawing on those relationships in future study. This applies within mathematics, so that students can translate readily between fractions and decimals, or between algebra and geometry; to other content areas, so that students understand how mathematics is used in the sciences, the social sciences, and the arts; and to the everyday world, so that students can connect school mathematics to daily life.

**Reasoning.** Mathematical reasoning is the critical skill that enables a student to make use of all other mathematical skills. With the development of mathematical reasoning, students recognize that mathematics makes sense and can be understood. They learn how to evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how those solutions can be applied.

**Representations.** Representations refer to the use of physical objects, drawings, charts, graphs, and symbols to represent mathematical concepts and problem situations. By using various representations, students will be better able to communicate their thinking and solve problems. Using multiple representations will enrich the problem solver with alternative perspectives on the problem. Historically, people have developed and successfully used manipulatives (concrete representations such as fingers, base ten blocks, geoboards, and algebra tiles) and other representations (such as coordinate systems) to help them understand and develop mathematics.

**Technology.** Calculators and computers need to be used along with other mathematical tools by students in both instructional and assessment activities. These tools should be used, not to replace mental math and paper-and-pencil computational skills, but to enhance understanding of mathematics and the power to use mathematics. Students should explore both new and familiar concepts with calculators and computers and should also

become proficient in using technology as it is used by adults (e.g., for assistance in solving real-world problems).

**At each grade level, with respect to content appropriate for that grade level, students will:**

**A. Problem Solving**

- Learn mathematics through problem solving, inquiry, and discovery.
- Solve problems that arise in mathematics and in other contexts (cf. workplace readiness standard 8.3).
  - Open-ended problems
  - Non-routine problems
  - Problems with multiple solutions
  - Problems that can be solved in several ways
- Select and apply a variety of appropriate problem-solving strategies (e.g., “try a simpler problem” or “make a diagram”) to solve problems.
- Pose problems of various types and levels of difficulty.
- Monitor their progress and reflect on the process of their problem solving activity.

**B. Communication**

- Use communication to organize and clarify their mathematical thinking.
  - Reading and writing
  - Discussion, listening, and questioning
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
- Analyze and evaluate the mathematical thinking and strategies of others.  
Use the language of mathematics to express mathematical ideas precisely.

**C. Connections**

- Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
- Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
- Recognize that mathematics is used in a variety of contexts outside of mathematics.
- Apply mathematics in practical situations and in other disciplines.
- Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

## D. Reasoning

- Recognize that mathematical facts, procedures, and claims must be justified.
- Use reasoning to support their mathematical conclusions and problem solutions.
- Select and use various types of reasoning and methods of proof.
- Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
- Make and investigate mathematical conjectures.
  - Counterexamples as a means of disproving conjectures
  - Verifying conjectures using informal reasoning or proofs.
- Evaluate examples of mathematical reasoning and determine whether they are valid.

## E. Representations

- Create and use representations to organize, record, and communicate mathematical ideas.
  - Concrete representations (e.g., base-ten blocks or algebra tiles)
  - Pictorial representations (e.g., diagrams, charts, or tables)
  - Symbolic representations (e.g., a formula)
  - Graphical representations (e.g., a line graph)
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

## F. Technology

- Use technology to gather, analyze, and communicate mathematical information.
- Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information (cf. workplace readiness standard 8.4-D).
- Use graphing calculators and computer software to investigate properties of functions and their graphs.
- Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
- Use computer software to make and verify conjectures about geometric objects.
- Use computer-based laboratory technology for mathematical applications in the sciences (cf. Science standards).

